

Logic

Exercise Sheet 1

Exercise 1.1 Does Wango semantle? – A motivation for logic

Calusians wingle.
Wango is Calusian.
Anyone who wingles semantles.

Does Wango semantle?

- a) Can you answer the question? Do you know what it means to *semantle*?
- b) Translate the text into the formal language of logic, using the connectives you know from previous courses.
- c) Can you imagine a computer answering the question?

Exercise 1.2 Setting up your environment

The following exercises deal with the simple arithmetic expressions defined in the introduction:

```
type expression =  
  Var of string  
  | Const of int  
  | Add of expression * expression  
  | Mul of expression * expression
```

You should have a working installation of OCaml, and a copy of the book code¹. At the OCaml prompt, load the complete book code with

```
#use "init.ml";;
```

Then, to have the right book chapter loaded, do a

```
#use "intro.ml";;
```

You can now enter arithmetic expressions and have them parsed for you:

```
<< 3 + x * (y + 1) >>
```

¹Note that the code needs to be built first using `make`. If you are running windows, then you might not have the `make` tool, and you can get around this by downloading http://www4.in.tum.de/~krauss/atp_interactive.ml and placing it in the code directory.

Exercise 1.3 Manipulating expressions in OCaml

- a) Write a function `eval` that evaluates expressions for a given valuation, which is a function mapping variable names to values:

$$eval : (string \rightarrow int) \rightarrow expression \rightarrow int$$

- b) Write a function `subst` that substitutes another term for a variable:

$$subst : string \rightarrow expression \rightarrow expression \rightarrow expression$$

- c) Create an extended expression type `mexpression` that adds a constructor `Sub` for subtraction. Also extend your evaluation function `meval` to the new type.

Now write a function

$$convert : mexpression \rightarrow expression$$

that eliminates the new construct expressing it in terms of the other ones. Make sure that

$$eval\ env\ (convert\ exp) = meval\ env\ exp$$

holds.

Exercise 1.4 Canonical Forms

Write code that “multiplies out” an expression by applying distributivity, and then reorders the terms to a canonical form.

You can use this code to check if two expressions are equivalent.