

# Logic

## Exercise Sheet 10

### Exercise 10.1 Warm-up: Miniscoping

Transform the following formulas such that they fall into the AE-fragment:

- a)  $\exists x.\forall y.P(x) \vee (Q(y) \wedge \neg Q(x))$
- b)  $\forall x.\exists y.\forall z.P(z) \wedge P(y) \vee \neg Q(z) \wedge Q(x)$

### Exercise 10.2 FOL with only one variable

We consider formulas without function symbols, where the only variable name that is allowed is  $x$ . (Example:  $\forall x.\exists x.P(x, x) \vee Q(x)$ ).

Show that this class is decidable.

### Exercise 10.3 Finite models for EA

Consider a formula in the EA-Fragment, i.e., of the form  $p = \exists x_1, \dots, x_n.\forall y_1, \dots, y_m.q$ , where  $q$  is quantifier-free and without function symbols.

Show that  $p$  has a model iff it has a finite model of size  $n$  (or 1 in the case  $n = 0$ ).

### Exercise 10.4 Strictly positive formulas

Find a simple procedure for deciding validity of formulas built only from  $\forall, \exists, \wedge, \vee, \top, \perp$ , where function symbols are allowed.

### Exercise 10.5

Implement the decision procedure you developed in Ex. 10.2.