

Logic

Exercise Sheet 13

Exercise 13.1 $\text{Th}(\mathbb{R}, 0, 1, +, -, \min, \max, <, \leq, =)$

- a) Show that quantifier elimination for real linear arithmetic still works if we add the binary functions \min and \max .
- b) Use the elimination to prove the following formula:

$$\forall abcd. \max(a, b) < \min(c, d) \Rightarrow ((\forall x. x > 2a \vee x \leq a) \vee (\exists y. 0 \leq y \wedge \max(c, y) = c))$$

- c) Show that we can also add the function $|\cdot|$ that yields the absolute value of its argument.

Exercise 13.2 Induction

Use induction to show that there is no quantifier-free formula with just $\wedge, \vee, \leq, =$ that is equivalent to $x < y$.

Exercise 13.3 Independence

A set Γ of propositional formulas is called *independent* if $p \in \Gamma$ implies $\Gamma \setminus \{p\} \not\models p$.

Show that any finite set has an equivalent independent subset.

Exercise 13.4 Brainstorming

Collect about 30 important pieces of terminology that were used in the lecture and that you want to remember for the exam.

Also collect the three results that you find most interesting.