

Logic

Exercise Sheet 6

Exercise 6.1 Validity and satisfiability

Which of the following formulas is valid, which is satisfiable?

For each satisfiable formula give a model. For each formula that is satisfiable but not valid, give a counter-model.

- a) $(\forall x.\exists y.P(x, y)) \Rightarrow (\exists y.\forall x.P(x, y))$
- b) $(\exists x.\forall y.P(x, y)) \Rightarrow (\forall y.\exists x.P(x, y))$
- c) $(\exists x.\forall y.P(x, y)) \Rightarrow (\forall x.\exists y.P(x, y))$

Exercise 6.2 More validity, satisfiability and models

Prove or disprove, using the definitions:

- a) For each sentence p , either p is valid or $\neg p$ is valid.
- b) A sentence p is valid iff $\neg p$ is unsatisfiable.

Exercise 6.3 Trivial model

Prove that a formula containing only $\wedge, \vee, \forall, \exists, \Rightarrow$ and atomic formulas is always satisfiable.

Exercise 6.4 Extension with $\exists!$

- a) Extend the definition of first-order formulas with an additional quantifier $\exists!$, such that $\exists!x.P(x)$ means that there exists a unique x such that $P(x)$ holds. Modify the definition of the semantics.
- b) Extend the OCaml implementations of the type `formula` and the function `holds` accordingly.
- c) Is the following formula valid?

$$(\exists!x.\exists!y.P(x, y)) \Rightarrow (\exists!y.\exists!x.P(x, y))$$