

Logic

Exercise Sheet 8

Exercise 8.1 Warm-Up

a) For each of the following formulas, give the domain of the Herbrand model.

i) $P(x) \Rightarrow P(c)$ ii) $P(x) \Rightarrow Q(f(x), g(c))$ iii) $\forall x. \exists y. P(x, y)$

b) Which of the following unification problems are solvable? Give the most general unifier if it exists.

i) $f(x, y) =? f(h(a), x)$ iii) $f(x, b) =? f(h(y), z)$
ii) $f(x, y) =? f(h(x), x)$ iv) $f(x, x) =? f(h(y), y)$

Exercise 8.2 Herbrand Model

Prove the following formula by skolemizing its negation, and finding a set of ground instances in the Herbrand model that is propositionally unsatisfiable:

$$(\forall x. P(x, f(x))) \Rightarrow (\exists y. P(c, y))$$

Exercise 8.3 Resolution

Formalize the following propositions in first-order logic and use resolution to show that a) implies b):

- a) Professor p is happy if all his students like logic.
- b) Professor p is happy if he has no students.

Exercise 8.4 Exponential Behaviour

Show that the time complexity of our unification algorithm is exponential.

Hint: construct a unification problem $s_1 =? t_1, \dots, s_n =? t_n$, such that the terms grow exponentially when applying the most general unifier.

Exercise 8.5 Unification

The code for unification in the book has some optimizations to improve its space usage. Develop a straightforward OCaml implementation of unification as it was presented in the lecture.