Equivalence of big and small steps

Method bodies and local variables

Local variables for execution of method body in method call:

- Big step semantics:
  only *this* and the parameters
- Small step semantics:
  also access to all enclosing local variables

  $$\{V{:}T;\ addr\ a.M([Val\ v])\} \to$$
  $$\{V{:}T;\ \{P{:}T';\ P := Val\ v;\ e\}\}$$

  if M has method body $([P], e)$

$\Longrightarrow$ dynamic binding — more bug than feature

Method bodies should only access *this* and the parameters

Free variables (= global variables)

$$fv :: expr \Rightarrow vname\ set$$

$fv\ (\texttt{Cast}\ C\ e) = fv\ e$

$fv\ (\texttt{Val}\ v) = \emptyset$

$fv\ (e_1\ «bop»\ e_2) = fv\ e_1 \cup fv\ e_2$

$fv\ (\texttt{Var}\ V) = \{V\}$

$fv\ (V := e) = \{V\} \cup fv\ e$

$fv\ (e.F\{D\}) = fv\ e$

$fv\ \{V{:}T;\ e\} = fv\ e - \{V\}$

$fv\ (\texttt{try}\ e_1\ \texttt{catch}\ (C\ V)\ e_2) = fv\ e_1 \cup (fv\ e_2 - \{V\})$

$\vdots$

Weak well-formedness

Definition A program is *weakly well-formed* (*wwf-J-prog P*) iff every method body (Vs, e) uses only its own local variables, *this* and the parameters *Vs*:

$$fv\ e \subseteq \{this\} \cup set\ Vs$$

Equivalence

Theorem If *wwf-J-prog P* then
$P \vdash \langle e,s \rangle \Rightarrow \langle e',s' \rangle$ iff $(P \vdash \langle e,s \rangle \rightarrow^* \langle e',s' \rangle \wedge \mathit{final}\ e')$

Proof of $\Longrightarrow$ and $\Longleftarrow$ separately.

Big $\Longrightarrow$ *small*

Theorem If *wwf-J-prog P* and $P \vdash \langle e,s \rangle \Rightarrow \langle e',s' \rangle$ then $P \vdash \langle e,s \rangle \rightarrow^* \langle e',s' \rangle$.

Proof by rule induction on $\Rightarrow$. Only blocks, try-catch and (especially!) method call are non-trivial.

small $\Longrightarrow$ *Big*

Theorem If *wwf-J-prog P* and $P \vdash \langle e,s \rangle \to^* \langle e',s' \rangle$ and *final e'* then $P \vdash \langle e,s \rangle \Rightarrow \langle e',s' \rangle$.

Proof by induction on $\to^*$.

- Basis: final expressions evaluate to themselves
- Step: by Lemma

Lemma If *wwf-J-prog P* and $P \vdash \langle e,s \rangle \to \langle e'',s'' \rangle$ and $P \vdash \langle e'',s'' \rangle \Rightarrow \langle e',s' \rangle$ then $P \vdash \langle e,s \rangle \Rightarrow \langle e',s' \rangle$.

Proof by rule induction on $\to$.