Small Step Semantics

Transition format

$$P \vdash \langle e,s \rangle \rightarrow \langle e',s' \rangle$$

$$P \vdash \langle es,s \rangle \;[\rightarrow]\; \langle es',s' \rangle$$

Reduction strategy:

> Reduce subexpressions as long as necessary, then reduce whole expression.

Two kinds of rules

- Subexpression reduction (for construct c):

  $P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
  $P \vdash \langle c \ldots\; e \ldots,\; s\rangle \to \langle c \ldots\; e' \ldots,\; s'\rangle$

  Example:

  $$P \vdash \langle e,s\rangle \longrightarrow \langle e',s'\rangle \Longrightarrow$$

  $$P \vdash \langle \texttt{Cast}\; C\; e,s\rangle \longrightarrow \langle \texttt{Cast}\; C\; e',s'\rangle$$

- Expression reduction

  Example: $P \vdash \langle \texttt{Cast}\; C\; null,s\rangle \to \langle null,s\rangle$

Notation: state components

$$hp \equiv fst$$
$$lcl \equiv snd$$

Val and Var

$$\frac{lcl\ s\ V = \lfloor v \rfloor}{P \vdash \langle \texttt{Var}\ V, s \rangle \rightarrow \langle \texttt{Val}\ v, s \rangle}$$

Field access

$$\frac{P \vdash \langle e,s\rangle \to \langle e',s'\rangle}{P \vdash \langle e.F\{D\},s\rangle \to \langle e'.F\{D\},s'\rangle}$$

$$\frac{hp\ s\ a = \lfloor(C,\ fs)\rfloor \qquad fs\ (F,\ D) = \lfloor v\rfloor}{P \vdash \langle addr\ a.F\{D\},s\rangle \to \langle \mathtt{Val}\ v,s\rangle}$$

Binary operations

$$\frac{P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle}{P \vdash \langle e \text{ «}bop\text{» } e_2,s\rangle \rightarrow \langle e' \text{ «}bop\text{» } e_2,s'\rangle}$$

$$\frac{P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle}{P \vdash \langle \texttt{Val}\ v_1 \text{ «}bop\text{» } e,s\rangle \rightarrow \langle \texttt{Val}\ v_1 \text{ «}bop\text{» } e',s'\rangle}$$

$$\frac{binop\ (bop,\ v_1,\ v_2) = \lfloor v \rfloor}{P \vdash \langle \texttt{Val}\ v_1 \text{ «}bop\text{» } \texttt{Val}\ v_2,s\rangle \rightarrow \langle \texttt{Val}\ v,s\rangle}$$

new

$$\frac{\begin{array}{c} \mathit{new\text{-}Addr}\ h = \lfloor a \rfloor \\ P \vdash C\ \textbf{has-fields}\ \mathit{FDTs} \\ h' = h(a \mapsto (C, \mathit{init\text{-}fields}\ \mathit{FDTs})) \end{array}}{P \vdash \langle \texttt{new}\ C,(h, l)\rangle \to \langle \mathit{addr}\ a,(h', l)\rangle}$$

Cast

$$\frac{P \vdash \langle e,s\rangle \to \langle e',s'\rangle}{P \vdash \langle \texttt{Cast}\ C\ e,s\rangle \to \langle \texttt{Cast}\ C\ e',s'\rangle}$$

$$\frac{hp\ s\ a = \lfloor(D,\ fs)\rfloor \qquad P \vdash D \preceq^* C}{P \vdash \langle \texttt{Cast}\ C\ (addr\ a),s\rangle \to \langle addr\ a,s\rangle}$$

$$P \vdash \langle \texttt{Cast}\ C\ null,s\rangle \to \langle null,s\rangle$$

Variable assignment

$$\frac{P \vdash \langle e, s \rangle \to \langle e', s' \rangle}{P \vdash \langle V := e, s \rangle \to \langle V := e', s' \rangle}$$

$$P \vdash \langle V := \texttt{Val}\ v, (h, l) \rangle \to \langle \texttt{Val}\ v, (h, l(V \mapsto v)) \rangle$$

Field assignment

$$\frac{P \vdash \langle e,s\rangle \to \langle e',s'\rangle}{P \vdash \langle e.F\{D\} \texttt{:=} e_2,s\rangle \to \langle e'.F\{D\} \texttt{:=} e_2,s'\rangle}$$

$$\frac{P \vdash \langle e,s\rangle \to \langle e',s'\rangle}{P \vdash \langle \texttt{Val}\ v.F\{D\} \texttt{:=} e,s\rangle \to \langle \texttt{Val}\ v.F\{D\} \texttt{:=} e',s'\rangle}$$

$$\frac{\begin{array}{c} h\ a = \lfloor(C,\ fs)\rfloor \\ fs' = fs((F,\ D) \mapsto v) \qquad h' = h(a \mapsto (C,\ fs')) \end{array}}{P \vdash \langle addr\ a.F\{D\} \texttt{:=} \texttt{Val}\ v,(h,\ l)\rangle \to \langle \texttt{Val}\ v,(h',\ l)\rangle}$$

Method call

$$\frac{P \vdash \langle e,s \rangle \to \langle e',s' \rangle}{P \vdash \langle e.M(es),s \rangle \to \langle e'.M(es),s' \rangle}$$

$$\frac{P \vdash \langle es,s \rangle \ [\to] \ \langle es',s' \rangle}{P \vdash \langle \texttt{Val}\ v.M(es),s \rangle \to \langle \texttt{Val}\ v.M(es'),s' \rangle}$$

$$\llbracket hp\ s\ a = \lfloor (C,\ fs) \rfloor;\ P \vdash C\ \textbf{sees}\ M\colon Ts \longrightarrow T = (pns,\ body)\ \textbf{in}\ D;$$
$$|vs| = |pns|;\ |Ts| = |pns| \rrbracket$$
$$\Longrightarrow P \vdash \langle addr\ a.M(map\ \texttt{Val}\ vs),s \rangle \longrightarrow$$
$$\langle blocks\ (this{\cdot}pns,\ Class\ D{\cdot}Ts,\ Addr\ a{\cdot}vs,\ body),s \rangle$$

$$blocks([V,\ldots],[T,\ldots],[v,\ldots],e) = \{V{:}T;\ V := Val\ v;\ \ldots\ e\}$$

Evaluate *es* from left to right:

$$\frac{P \vdash \langle e,s \rangle \rightarrow \langle e',s' \rangle}{P \vdash \langle e \cdot es,s \rangle \; [\rightarrow] \; \langle e' \cdot es,s' \rangle}$$

$$\frac{P \vdash \langle es,s \rangle \; [\rightarrow] \; \langle es',s' \rangle}{P \vdash \langle \texttt{Val}\ v \cdot es,s \rangle \; [\rightarrow] \; \langle \texttt{Val}\ v \cdot es',s' \rangle}$$

Local variable

Trick: store value of local variable as $\{V{:}T;\ V\ \texttt{:=}\ \texttt{Val}\ v\texttt{;}\ e'\}$

assigned $V\ e \equiv \exists v\ e'.\ e = V\ \texttt{:=}\ \texttt{Val}\ v\texttt{;}\ e'$

$\llbracket P \vdash \langle e,(h, l(V := None))\rangle \rightarrow \langle e',(h', l')\rangle;\ l'\ V = None;$

$\neg$ *assigned* $V\ e\rrbracket$

$\Longrightarrow P \vdash \langle\{V{:}T;\ e\},(h, l)\rangle \rightarrow \langle\{V{:}T;\ e'\},(h', l'(V := l\ V))\rangle$

$\llbracket P \vdash \langle e,(h, l(V := None))\rangle \rightarrow \langle e',(h', l')\rangle;\ l'\ V = \lfloor v\rfloor;$

$\neg$ *assigned* $V\ e\rrbracket$

$\Longrightarrow P \vdash \langle\{V{:}T;\ e\},(h, l)\rangle \rightarrow$

$\langle\{V{:}T;\ V\ \texttt{:=}\ \texttt{Val}\ v\texttt{;}\ e'\},(h', l'(V := l\ V))\rangle$

Local variable (2)

$$[\![P \vdash \langle e,(h, l(V \mapsto v))\rangle \rightarrow \langle e',(h', l')\rangle;\ l'\ V = \lfloor v' \rfloor]\!]$$

$$\Longrightarrow P \vdash \langle\{V{:}T;\ V \texttt{ := Val } v\texttt{; } e\},(h, l)\rangle \rightarrow$$

$$\langle\{V{:}T;\ V \texttt{ := Val } v'\texttt{; } e'\},(h', l'(V := l\ V))\rangle$$

$$P \vdash \langle\{V{:}T;\ V \texttt{ := Val } v\texttt{; Val } u\},s\rangle \rightarrow \langle \texttt{Val } u,s\rangle$$

$$P \vdash \langle\{V{:}T; \texttt{Val } u\},s\rangle \rightarrow \langle \texttt{Val } u,s\rangle$$

Sequential composition

$$\frac{P \vdash \langle e, s\rangle \rightarrow \langle e', s'\rangle}{P \vdash \langle e\texttt{;}\ e_2, s\rangle \rightarrow \langle e'\texttt{;}\ e_2, s'\rangle}$$

$$P \vdash \langle \texttt{Val}\ v\texttt{;}\ e_2, s\rangle \rightarrow \langle e_2, s\rangle$$

Conditional

$$\frac{P \vdash \langle e, s\rangle \rightarrow \langle e', s'\rangle}{P \vdash \langle \texttt{if}\ (e)\ e_1\ \texttt{else}\ e_2, s\rangle \rightarrow \langle \texttt{if}\ (e')\ e_1\ \texttt{else}\ e_2, s'\rangle}$$

$$P \vdash \langle \texttt{if}\ (\mathit{true})\ e_1\ \texttt{else}\ e_2, s\rangle \rightarrow \langle e_1, s\rangle$$
$$P \vdash \langle \texttt{if}\ (\mathit{false})\ e_1\ \texttt{else}\ e_2, s\rangle \rightarrow \langle e_2, s\rangle$$

While loop

$$P \vdash \langle \texttt{while}\,(b)\; c, s\rangle \rightarrow$$
$$\langle \texttt{if}\,(b)\,(c\texttt{;}\ \texttt{while}\,(b)\; c)\;\texttt{else}\; \mathit{unit}, s\rangle$$

Example

$$P \vdash \langle \{ \mathit{V}{:}\mathit{Boolean};\ \mathit{U}\ \texttt{:=}\ \mathit{true}\texttt{;}\ \mathit{V}\ \texttt{:=}\ \texttt{Var}\ \mathit{U}\texttt{;}\ \texttt{Var}\ \mathit{V} \},$$

$$(\mathit{h},\ \mathit{empty})\rangle \longrightarrow^{*}$$

$$\langle \mathit{true},(\mathit{h},\ [\mathit{U} \mapsto \mathit{Bool\ True}])\rangle$$

Example

```
class C extends Object
{ field F:Integer
  method m(i:Integer):Integer =
    this.F{C} + Var i
}
```

State:

$h = [a \mapsto (C, fs)]$

$fs = [(F, C) \mapsto \mathit{Intg}\ 2]$

$l = [V \mapsto \mathit{Addr}\ a]$

Reduction:

$$P \vdash \langle \texttt{Var}\ V.m([\texttt{Val}\ (\mathit{Intg}\ 1)]),(h, l)\rangle \rightarrow^* \langle \texttt{Val}\ (\mathit{Intg}\ 3),(h, l)\rangle$$