



0. Quantum
Compiler

I. Quantum
Information

II. Quantum Control

III. Local Control

Conclusions &
Outlook

Quantum Computing as an Engineering Problem: An Optimal-Control-Based Quantum Compiler

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Seminar “Pearls in Computer Science”, TUM, April 2007

Motivation: Control in Quantum Technology

*We are currently in the midst of **a second quantum revolution**. The first quantum revolution gave us new rules that govern physical reality. The second quantum revolution will take these rules and use them to develop **new technologies**.* DOWLING & MILBURN, 2003

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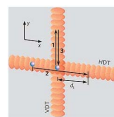
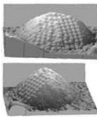
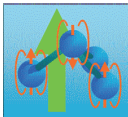
Conclusions & Outlook

- economy

currently some 30% of the GNP of industrial states depend on quantum effects (transistor, laser)

- technology ahead

quantum & nano-technology rely on **quantum control**
(solid-state devices, spintronics–NMR–EPR, quantum dots, ion-traps)



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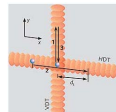
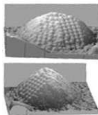
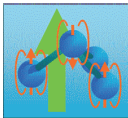
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Classical Compiler

Compilation into Machine Code

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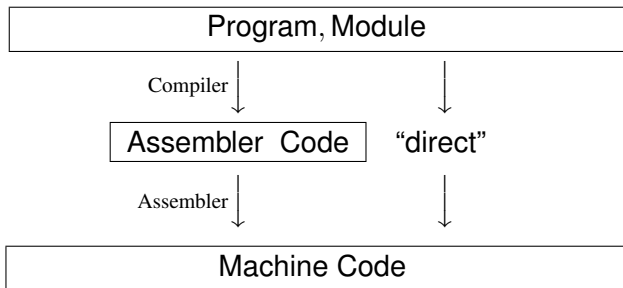
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■ Classical Compilation





Quantum Compiler

Compilation into Machine Code of Quantum Controls

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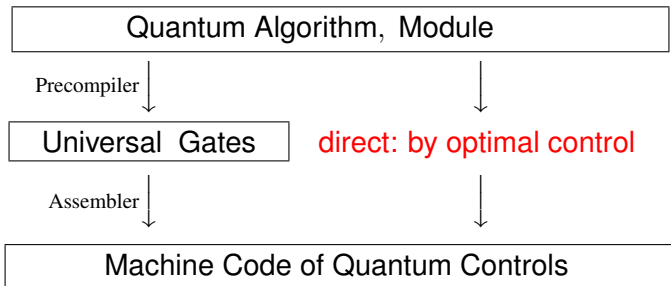
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■ Quantum Compilation: a Control Problem



crucial for quantum machine code:
has to be timeoptimal or decoherence protected



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Compilation into Machine Code of Quantum Controls

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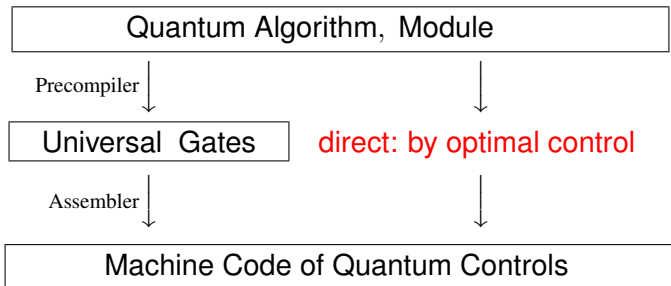
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Outline

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- 1 Quantum Compilation
- 2 First Glimpses into the Quantum World
 - Quantum Basics
 - Quantum Computing
- 3 Quantum Compilation by Optimal Control
 - Principles and Tasks
 - Gradient-Flow Algorithms
 - Timeoptimal Control
 - Decoherence Control
- 4 Local Control
- 5 Conclusions & Outlook



What is Special with Quantum Systems?

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Fascinating Fact: unparalleled in classical physics

Theorem

*Generically, a multi-partite quantum system contains **more information** than is accessible from piecing together the information of its components.*

Corollary

“The total is more than the sum of its constituent pieces.”



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Quantum Systems – Basic Notions and Notations: “Qubits”

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- 1 representation:** *pure* quantum states are represented by vectors, e.g. in a two-level system
 $|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{C}^2$ and $|1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in \mathbb{C}^2$
- 2 superposition:** linear combinations give new states
 $\cos \phi |0\rangle + \sin \phi |1\rangle$
- 3 composition:** take tensor product of constituents
 $|01\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (0, 1, 0, 0)^t \in \mathbb{C}^4$
- 4** the state space forms a **Hilbert space**,
in which the basis $\{|0\rangle, |1\rangle\}$ is orthonormal
 $\langle 0|1\rangle = \langle 1|0\rangle = 0$ (where $\langle 0| := (10)$)
 $\langle 0|0\rangle = \langle 1|1\rangle = 1$ (note $|0\rangle\langle 0| := (10)^t(10) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$)
- 5 general quantum states:** combine dyadic products

$$\rho = \sum_{ij} \lambda_{ij} |\psi_i\rangle\langle\psi_j|$$

with $\lambda_{ij} = \lambda_{ji}^*$ and $\text{tr } \rho = \sum_j \lambda_{jj} = 1$ and $\rho \geq 0$



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Information Content of Quantum Systems

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Example

Consider the Bell state $|\Phi\rangle := \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$:

$$\rho_{ab} = |\Phi\rangle\langle\Phi| = \frac{1}{2}(|00\rangle + |11\rangle)(\langle 00| + \langle 11|) \quad \text{(total info)}$$

$$\rho_a = \text{tr}_b |\Phi\rangle\langle\Phi| \quad \text{(local info a)}$$

$$= \frac{1}{2} \text{tr}_b (|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|)$$

$$= \frac{1}{2} (|0\rangle\langle 0|_a \langle 0|_b \langle 0|_b + |0\rangle\langle 1|_a \langle 1|_b \langle 0|_b + |1\rangle\langle 0|_a \langle 0|_b \langle 1|_b + |1\rangle\langle 1|_a \langle 1|_b \langle 1|_b)$$

$$= \frac{1}{2} (|0\rangle\langle 0|_a + |1\rangle\langle 1|_a)$$

$$= \frac{1}{2} \mathbf{1}_a$$

$$\rho_b = \text{tr}_a |\Phi\rangle\langle\Phi| = \frac{1}{2} \mathbf{1}_b \quad \text{(local info b)}$$



Quantum Information:

"The Total is More than the Sum of its Constituent Pieces"

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Proof (simplification).

The Example is generic: compare its information content

- 1 $\rho_{ab} = \rho_{ab}^2$ contains *all information* about **total system**;
- 2 $\rho_{a(b)}$ contains all information about **subsystems $a(b)$** ;
- 3 **reconstruction** $\rho'_{ab} := \rho_a \otimes \rho_b \neq \rho_{ab}^2$ puts together *all information accessible from both local subsystems*;
this is generically less than in ρ_{ab} .





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Remark: The reconstruction $\rho'_{ab} = \rho_a \otimes \rho_b = \frac{1}{4} \mathbb{1}$ is *diagonal*, whereas the original $\rho_{ab} = \sum_{\mu\nu} \lambda_{\mu\nu} (|a_\mu\rangle\langle a_\nu| \otimes |b_\mu\rangle\langle b_\nu|)$ contained *off-diagonal* terms. The **coherent phase relation** between the local constituents cannot be reconstructed *a posteriori* from *local* pieces of information.



Quantum Information: Entanglement

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Definition (Pure-State Entanglement)

An n -party pure quantum state that can be written as a **tensor product**

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$$

is called **product state**. If it cannot, it is called **entangled**.

Corollary

*Product states contain no more than local information, while **entangled states comprise more information than locally reconstructable**.*



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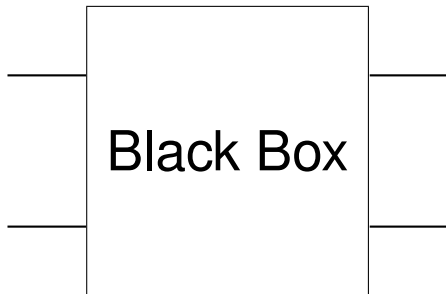
*Product states contain no more than local information, while **entangled states comprise more information than locally reconstructable**.*



Quantum Systems – as a New Computational Resource

Principle: for cracking a black box in one shot

- input : product of superpositions
- output: superposition of products (entangled)



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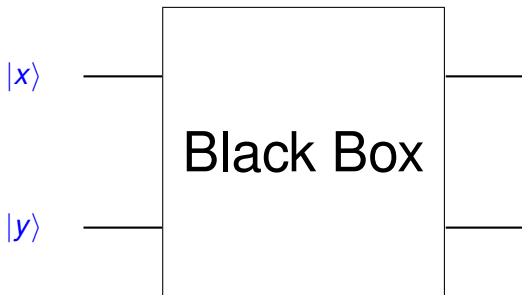
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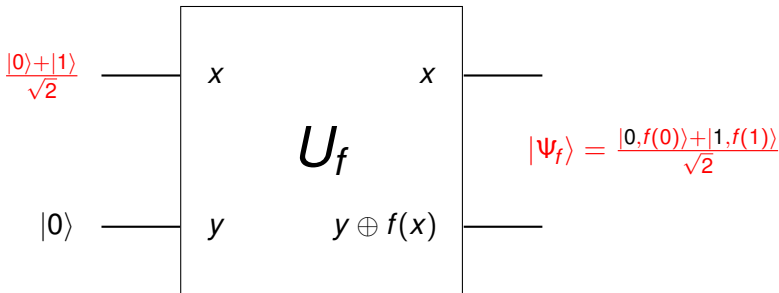
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Quantum Systems – as a New Computational Resource

Example



superposition of $|0\rangle, |1\rangle \mapsto$ superposition of $f(0), f(1)$

$f(0) = f(1)$ ('**constant**') \Rightarrow output '**product state**'

$f(0) \neq f(1)$ ('**balanced**') \Rightarrow output '**entangled state**'



Hadamard's Gate

- Auxiliary Single-Qubit Gate H :

$$|0\rangle \xrightarrow{H} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad (\text{symmetric})$$

$$|1\rangle \xrightarrow{H} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \quad (\text{anti-symmetric})$$

$$\text{thus } H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

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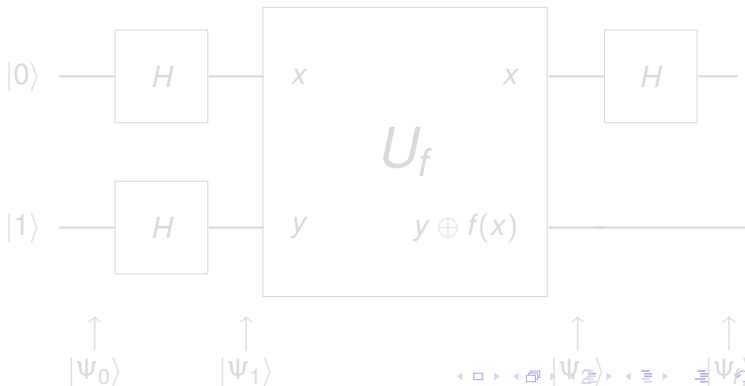
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Deutsch's Algorithm

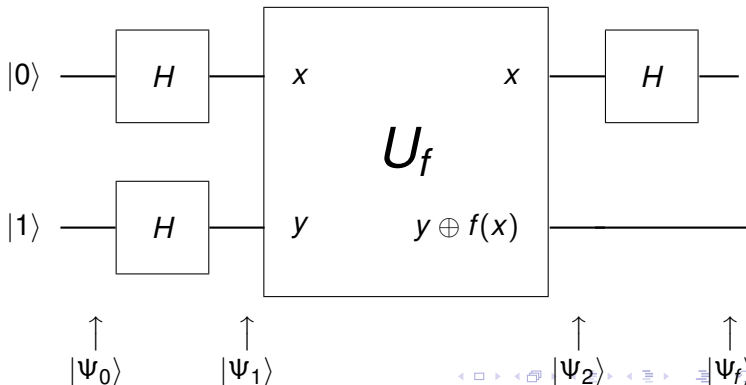
- **Task:** Decide whether **unknown** 'black-box function'
 $f : \{0, 1\} \rightarrow \{0, 1\}$ with $x \mapsto f(x)$
is **constant:** $f(0) = f(1)$, or **balanced:** $f(0) \neq f(1)$.





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Deutsch's Algorithm, Ctd

$$\blacksquare |\Psi_0\rangle \xrightarrow{H_{1,2}} |\Psi_1\rangle \xrightarrow{U_f} |\Psi_2\rangle \xrightarrow{H_1} |\Psi_f\rangle$$

$$|\Psi_0\rangle = |0\rangle_x |1\rangle_y$$

$$|\Psi_1\rangle = \frac{|0\rangle_x + |1\rangle_x}{\sqrt{2}} \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}}$$

$$|\Psi_2\rangle = \begin{cases} \pm \frac{|0\rangle_x + |1\rangle_x}{\sqrt{2}} \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}} & \text{for } f(0) = f(1) \\ \pm \frac{|0\rangle_x - |1\rangle_x}{\sqrt{2}} \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}} & \text{for } f(0) \neq f(1) \end{cases}$$

$$|\Psi_f\rangle = \pm |f(0) \oplus f(1)\rangle_x \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}}$$

$$\blacksquare \text{constant } f: f(0) \oplus f(1) = 0$$

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$$|\Psi_1\rangle = \frac{|0\rangle_x + |1\rangle_x}{\sqrt{2}} \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}}$$

$$|\Psi_2\rangle = \begin{cases} \pm \frac{|0\rangle_x + |1\rangle_x}{\sqrt{2}} \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}} & \text{for } f(0) = f(1) \\ \pm \frac{|0\rangle_x - |1\rangle_x}{\sqrt{2}} \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}} & \text{for } f(0) \neq f(1) \end{cases}$$

$$|\Psi_f\rangle = \pm |f(0) \oplus f(1)\rangle_x \frac{|0\rangle_y - |1\rangle_y}{\sqrt{2}}$$

■ **constant** f : $f(0) \oplus f(1) = 0$

■ **balanced** f : $f(0) \oplus f(1) = 1$



Deutsch's Algorithm, Ctd

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Hamiltonian Dynamics: Equations of Motion

States: $|\psi\rangle \in \mathcal{H}$, $\langle\psi|\psi\rangle=1$; operators: $H = H^\dagger \in \mathcal{B}(\mathcal{H})$,
 $\rho \in \mathcal{B}(\mathcal{H})$ with and $\rho \geq 0$, $\text{tr } \rho = 1$

■ pure quantum states: Schrödinger Equation

$$\begin{aligned} |\dot{\psi}(t)\rangle &= -iH |\psi(t)\rangle \\ |\psi(t)\rangle &= e^{-itH} |\psi(0)\rangle = U(t) |\psi(0)\rangle \end{aligned}$$

■ ensembles: Liouville-von Neumann Equation

$$\begin{aligned} \dot{\rho}(t) &= -i[H, \rho(t)] \\ \rho(t) &= U\rho(0)U^{-1} \end{aligned}$$

Hamiltonian dynamics ‘lives’ on **unitary orbit** of initial state.



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Hamiltonian Dynamics: Expectation Values

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Expectation value of observables $B = B^\dagger \in \mathcal{B}(\mathcal{H})$:

- pure quantum states:

$$\langle B \rangle := \langle \psi | B | \psi \rangle$$

- ensembles:

$$\text{tr}(B^\dagger \rho(t)) = \text{tr}(B^\dagger U \rho_0 U^{-1})$$



Hamiltonian Dynamics: Expectation Values

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■ Bilinear Control System

$$\dot{X}(t) = \left(A + \sum_{j=1}^m u_j(t) B_j \right) X(t)$$

■ Hamiltonian dynamics (Schrödinger equation)

$$|\dot{\psi}(t)\rangle = -i \left(H_d + \sum_{j=1}^m u_j(t) H_j \right) |\psi(t)\rangle$$

- H_d : drift term
- H_j : controls
- $u_j(t)$: control amplitudes



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Scope in Optimal Control:

maximise quality function **subject to** equation of motion

Scenarios:

■ Hamiltonian dynamics

notation: $U := e^{-itH}$; $\text{Ad}_U(\cdot) := U(\cdot)U^{-1}$; $\text{ad}_H(\cdot) := [H, \cdot]$

1. pure state $|\dot{\psi}\rangle = -iH |\psi\rangle \in \mathcal{H}$
2. gate $\dot{U} = -iH U \in \mathcal{U}(\mathcal{H})$
3. non-pure state $\dot{\rho} = -i \text{ad}_H(\rho) \in \mathcal{B}_1(\mathcal{H})$
4. projective gate $\dot{\text{Ad}}_U = -i \text{ad}_H \circ \text{Ad}_U \in \mathcal{U}(\mathcal{B}_1(\mathcal{H}))$

■ Master equations of dissipative dynamics

- 3'. non-pure state $\dot{\rho} = -(i \text{ad}_H + \Gamma)(\rho)$
- 4'. **contractive** map $\dot{\chi} = -(i \text{ad}_H + \Gamma) \circ \chi \in \mathcal{GL}(\mathcal{B}_1(\mathcal{H}))$



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Typical Tasks in Optimal Quantum Control

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1. Maximising Experimental Sensitivity:

maximise transfer amplitude $f := |\text{tr}\{C^\dagger UAU^\dagger\}|$,
subject to equation of motion

$$\dot{A} = -i\hat{H}A := -i(HA - AH)$$

Glaser, T.S.H., Sieveking, Schedletsky, Nielsen, Sørensen, Griesinger, *Science* **280** (1998), 421

2. Timeoptimal Quantum Compilation:

maximise fidelity $\text{Re tr}\{\hat{U}_G^\dagger \hat{U}(T)\}$

subject to equation of motion $\dot{\hat{U}}(t) = -i\hat{H} \circ \hat{U}(t)$

T.S.H., Spörl, Khaneja, Glaser, *Phys. Rev. A* **72** (2005), 042331

3. Quantum Compilation with Minimal Relaxative Loss:

maximise fidelity $\text{Re tr}\{\hat{U}_G^\dagger \chi(T)\}$

subject to equation of motion (now Master Equation)

$$\dot{\chi}(t) = -(i\hat{H} + \Gamma) \circ \chi(t)$$



Typical Tasks in Optimal Quantum Control

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[quant-ph/0609037](https://arxiv.org/abs/quant-ph/0609037)



Methods of Quantum Control

1. Gradient Flows on Unitary Group:

Principle: gradient flow on unitary group

- **General** $A, C \in \text{Mat}_n(\mathbb{C})$

set $f(U) := \text{tr}\{UAU^\dagger C\}$; $U = e^{-itH}$; $A_t := UAU^\dagger$
and $F_1(U) := \text{Re } f(U)$; and $F_2(U) := |f(U)|^2$ then

$$\text{grad-flow } \dot{U} = \text{grad } F_\nu(U) U = -G^{(\nu)} U$$

- recursive unitary transformations

$$U_{k+1}^{(\nu)} = e^{-\alpha_k^{(\nu)} G_k^{(\nu)}} U_k^{(\nu)} \quad \text{till} \quad \|G_{k+1}^{(\nu)}\|_2 \rightarrow 0$$

Science **280** (1998), 421 and *J. Global Optim.* **23** (2002), 283

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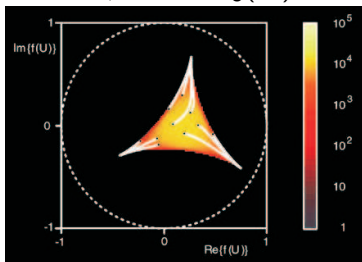
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Examples of Quantum Control

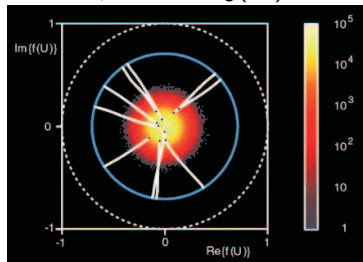
1. Maximising Experimental Sensitivity:

- find $r_C(A)$ by gradient flow on unitary group

$A, C \in \text{Mat}_3(\mathbb{C})$

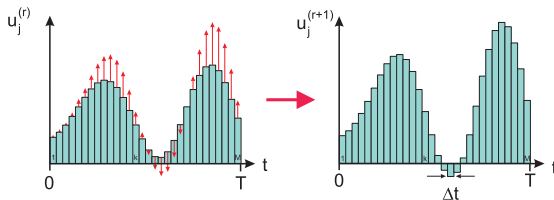


$A, C \in \text{Mat}_8(\mathbb{C})$



Glaser, T.S.H., Sieveking, Schedletzy, Nielsen, Sørensen, Griesinger, *Science* **280** (1998), 421

■ Gradient Ascent Algorithm GRAPE



J. Magn. Reson. **172** (2005), 296 and *Phys. Rev. A* **72** (2005), 042331

Generation of Unitary Operators

The diagram illustrates the generation of unitary operators. A curved arrow starts at $U(0) = 1$ and ends at $U(T)$. Below the arrow is the equation $\dot{U} = -i H U$.

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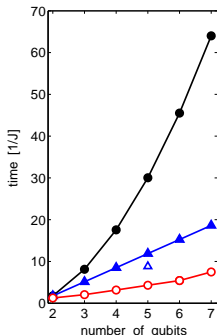
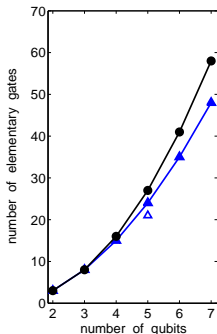


Examples of Quantum Control

2. Realising Quantum Gates in Minimal Time

Goal: *time complexity* of QFT on linear spin chain for

- Shor's algorithm
- all algorithms of (abelian) *hidden subgroup* type



⇒ timeopt. QFT: **some 3 times faster** than fastest current standard-gate decompositions

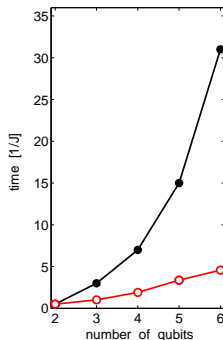
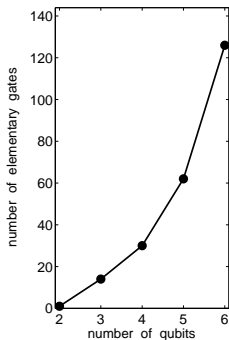


Examples of Quantum Control

2. Realising Quantum Gates in Minimal Time

Goal: *time complexity* of C^n NOT as module in

- Quantum Error Correction



⇒ timeopt. C^n NOT: **quadratically faster** than fastest current standard-gate decomposition

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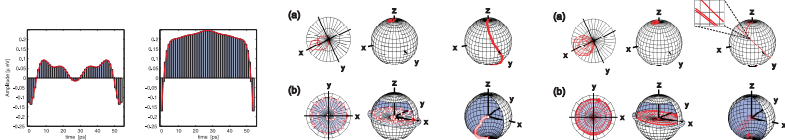
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2. Realising Quantum Gates in Minimal Time



⇒ timeopt. CNOT: **some 5 times faster** than NEC group

- Quality $q := Fe^{-\tau_{\text{op}}/\tau_Q}$

so $1 - q = 1 - 0.999999999 e^{-55\text{ps}/10\text{ns}} = \mathbf{0.0055}$

(NEC: $1 - q = 1 - 0.4188 e^{-250\text{ps}/10\text{ns}} = 0.5917$)

PRA 75, 012302 (2007)



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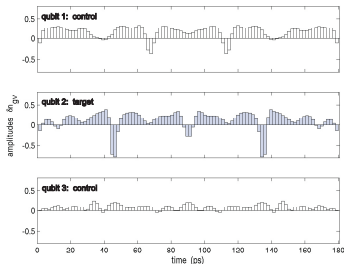
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Goal: TOFFOLI gate on 3 linearly coupled charge qubits



13 times faster than NEC

■ error rates cut by two orders of magnitude ($T_2 \simeq 10$ ns):

1 direct gate by optimal control

$$1 - q = 1 - 0.99999 e^{-180\text{ps}/10\text{ns}} = 0.0178$$

2 by 9 CNOT's from optimal control

$$1 - q = 1 - (0.999999999 e^{-55\text{ps}/10\text{ns}})^9 = 0.0483$$

3 by 9 CNOT's under pioneering controls

$$1 - q = 1 - (0.4188 e^{-250\text{ps}/10\text{ns}})^9 = 0.9997$$



Examples of Quantum Control

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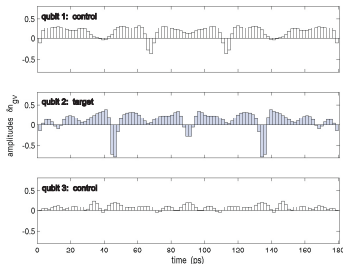
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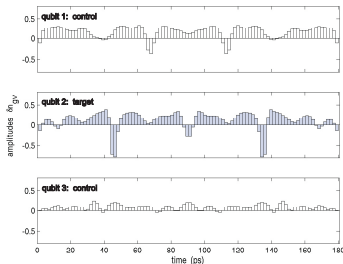
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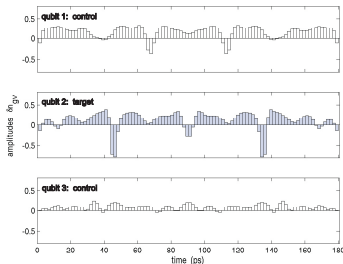
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Quantum Compiler

Compilation into Machine Code of Quantum Controls

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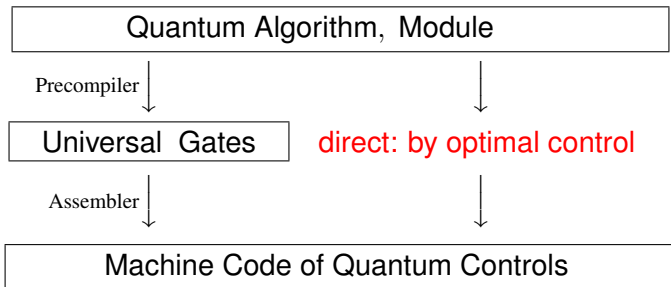
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■ Quantum Compilation: a Control Problem



crucial for **quantum** machine code:

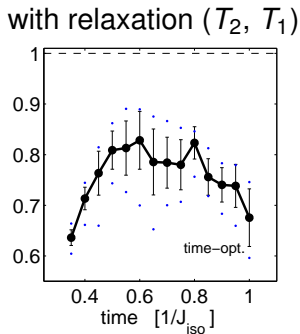
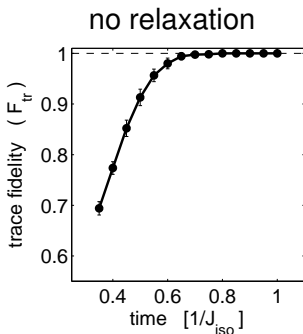
has to be **timeoptimal** or **decoherence protected**



Examples of Quantum Control

3. Decoherence Control: Results of System II

■ System-II: driving **outside** slowly-relaxing subspace



- mean of 15 time-optimised pulse sequences
- dissipation affects sequences differently

0. Quantum Compiler

I. Quantum Information

II. Quantum Control

Tasks

Gradient Algorithms

Timeoptimal Control

Decoherence Control

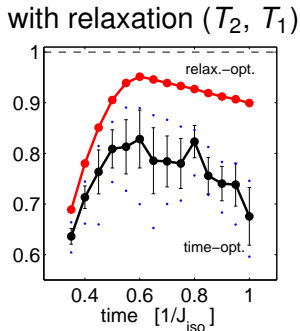
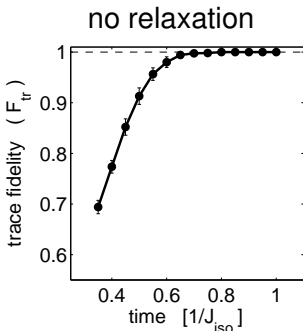
III. Local Control

Conclusions & Outlook

Examples of Quantum Control

3. Decoherence Control: Results of System II

■ System-II: driving **outside** slowly-relaxing subspace



- mean of 15 time-optimised pulse sequences
- dissipation affects sequences differently
- relaxation-optimised: **systematic substantial gain**

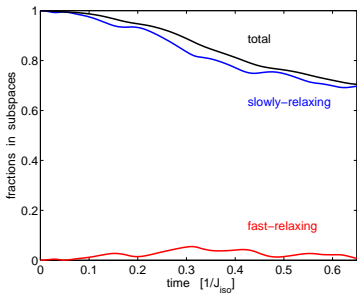


Examples of Quantum Control

3. Realising Quantum Gates with Minimal Relaxation

CNOT under **System-II**: Projection into Subspaces

■ time-optimal



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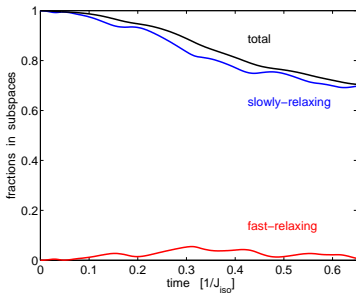


Examples of Quantum Control

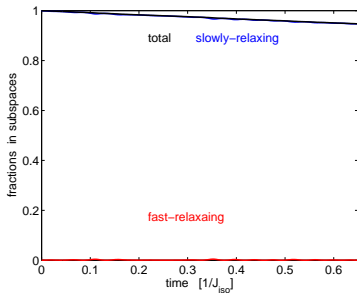
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■ opt. against decoherence



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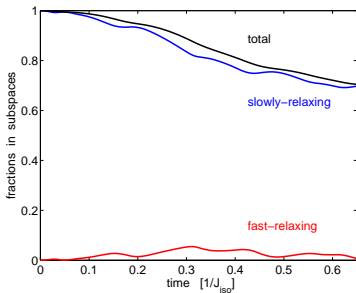
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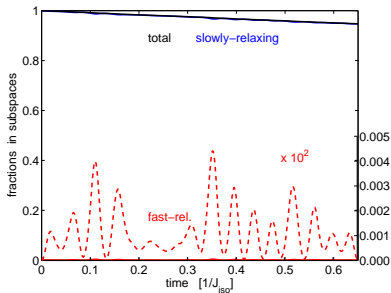
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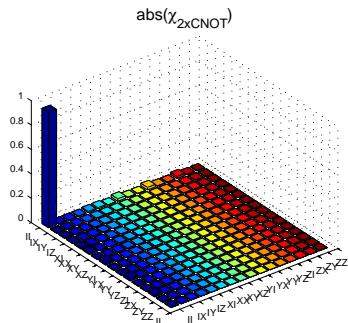
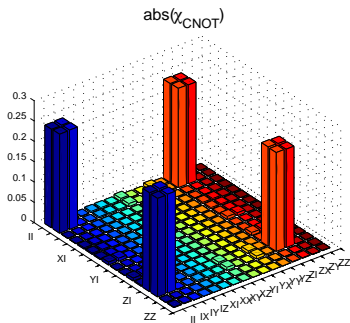
Timeoptimal Control

Decoherence Control

III. Local Control

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■ CNOT under **System-II**: Process Tomography of Gate Protected against Dissipation by Optimal Control





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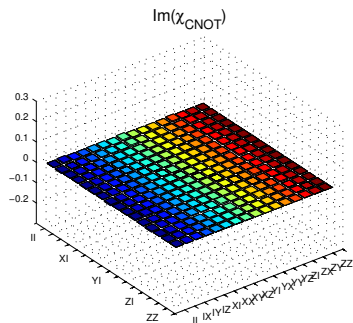
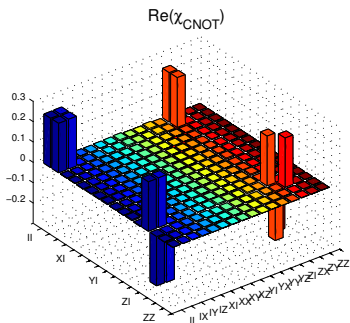
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Examples of Quantum Control

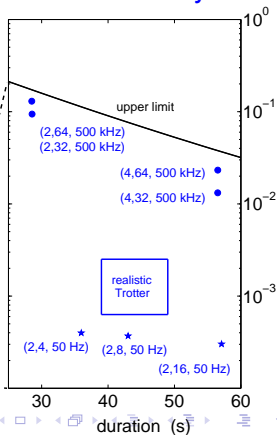
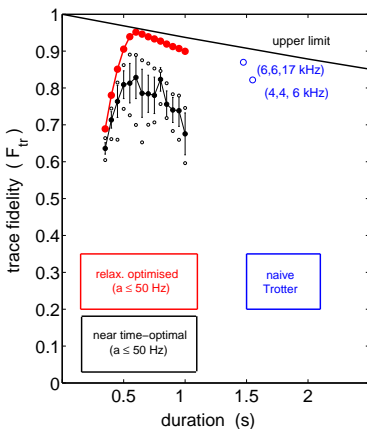
3. Realising Quantum Gates with Minimal Relaxation

quant-ph/0609037

■ CNOT under **System-II**: comparison of methods

by decoherence control:
> 95% fidelity

conventional:
< 15% fidelity



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Decoherence Control: Take-Home Message

Which Tool in Which Setting?

1. *“anything goes”*: Paul FEYERABEND
only in **ideal** case: decoherence-free,
fully **controllable and closed** under drift
2. *Timeoptimal Control*:
whenever **slowly**-relaxing subsystem **controllable
and closed** under drift
3. *Relaxation-Optimised Control*:
whenever **slowly**-relaxing subsystem **open**, where
subsystem
(i) **controllable** or
(ii) **to be extended** for controllability



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Topic III: Time-Reversal by Local Operations

Statement of the Problem

0. Quantum Compiler

I. Quantum Information

II. Quantum Control

III. Local Control

Conclusions & Outlook

Question:

Which quantum evolutions are reversible by **local** unitary operations ?

- Problem: given a Hamiltonian H and a time $t > 0$,
 $? \exists \{K_1, K_2\} \subset SU(2)^{\otimes n} : K_1 e^{-itH} K_2 = e^{+itH}$
- Equivalent to Problem: given a Hamiltonian H ,
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Local Time-Reversal

Case Distinction

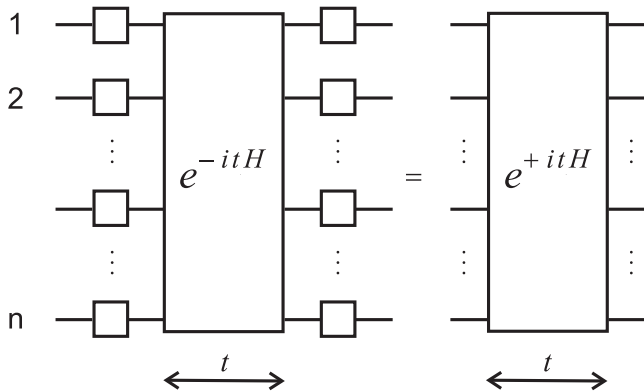
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Conclusions & Outlook



$$\text{Type-I: } K \cdot U(t) \cdot K^{-1} = U(-t) = U(t)^{-1}$$

$$\text{Type-II: } K_1 \cdot U(\tau) \cdot K_2 = U(-\tau) = U(\tau)^{-1}$$

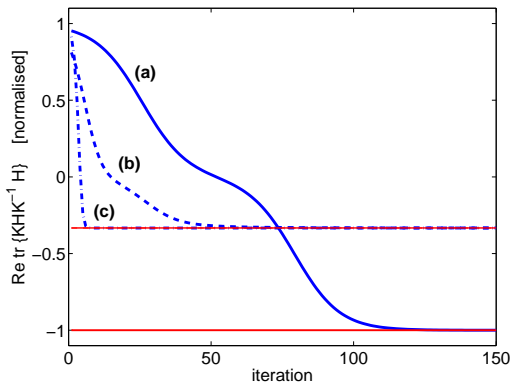


Local Time-Reversal

Examples of Local Gradient Flows

■ Examples

- (a) ISING ZZ-interaction on cyclic graph C_4 (bipartite)
- (b) ISING ZZ-interaction on cyclic graph C_3 (not bipartite)
- (c) HEISENBERG XXX interaction (isotropic coupling)



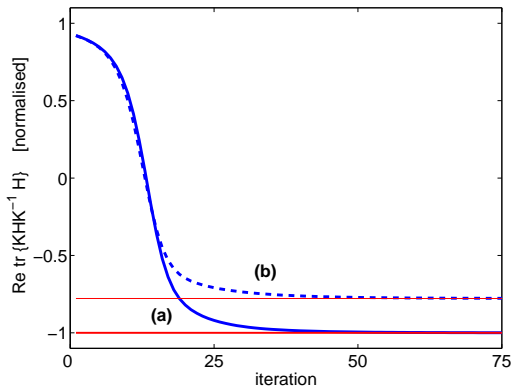


Local Time-Reversal

Examples of Local Gradient Flows

■ Examples

- (a) HEISENBERG XX -interaction on cyclic graph C_3 (not bipartite)
- (b) HEISENBERG $X(-X)$ -interaction on cyclic graph C_3





The Local C -Numerical Range

Local Quantum Control

with G. Dirr & U. Helmke

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Conclusions &
Outlook

Definition (math-ph/0701037 and math-ph/0702005)

The *local C -numerical range* is the set

$$W_{\text{loc}}(C, A) := \{\text{tr}(C^\dagger UAU^\dagger) \mid U \in SU(2)^{\otimes n}\} \subseteq W_C(A),$$

where the unitary orbit is restricted to *local operations*
 $U =: K \in SU(2) \otimes SU(2) \otimes \cdots \otimes SU(2)$



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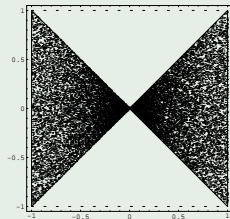
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Example (I non convex)

$$A := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1+i & 0 \\ 0 & 1-i \end{pmatrix}$$

$$C := \text{diag}(1, 0, 0, 0)$$





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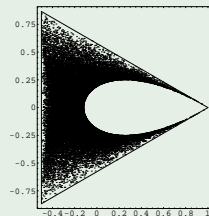
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Example (II neither star-shaped nor simply connected)

$$A := \begin{pmatrix} 1 & 0 \\ 0 & e^{2i\pi/3} \end{pmatrix}^{\otimes 3}$$

$$C := \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}^{\otimes 3}$$





The Local C -Numerical Range

Local Quantum Control

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Definition (math-ph/0701037 and math-ph/0702005)

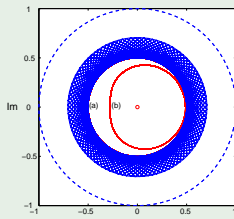
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Example (III distinct circular symmetry)

$$A := C := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$



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Applications

Pure-State Entanglement

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Conclusions & Outlook

- Maximising real part in $W_{\text{loc}}(C, A)$ minimises distance from C to *local unitary orbit* of A

$$\max_{K \in SU(2)^{\otimes n}} \text{Re tr}\{C^\dagger KAK^{-1}\} \Leftrightarrow \min_{K \in SU(2)^{\otimes n}} \|KAK^{-1} - C\|_2$$

- Application to Quantum Information Theory: let A be a given rank-1 state of the form $A = |\psi\rangle\langle\psi|$ and $C = \text{diag}(1, 0)^{\otimes n}$ [thus $W_{\text{loc}}(C, A) \rightarrow W_{\text{loc}}(A)$]

Corollary (Interpretation)

The minimal Euclidean distance is a measure of (pure-state) entanglement; i.e. it quantifies how far A is from the local equivalence class of the tensor-product state C .



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Pure-State Entanglement

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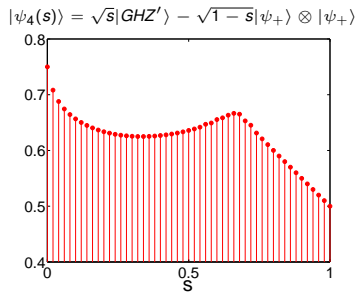
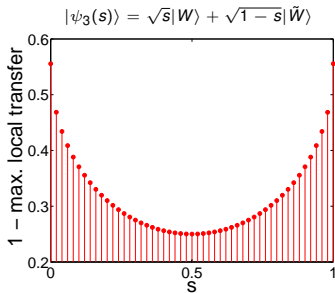
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Conclusions & Outlook

- Examples:
pure-state entanglement parameterised by s





Applications

Pure-State Entanglement

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Conclusions & Outlook

- CPU times: local gradient flows
very fast as compared to global techniques
- Example 1: distance to 3-qubit W state
- Example 2: distance to 4-qubit GHZ-type state

qubits	polynomial optim. cpu-time [sec] ¹	by gradient flow cpu-time [sec] ²	speed-up
3	10.92	0.30	36.4
4	103.97	0.71	147.0

¹ Eisert *et al.* (processor with 2.2 GHz, 1 GB RAM)

² average of 50 runs, Athlon XP1800+ (1.1 GHz, 512 MB RAM)



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Corollary (Extension I: Local C -Numerical Range)

For $H = H^\dagger$ with $\|H\|_2 = 1$ the following are equivalent:

- 1 the Hamiltonian H is locally sign-reversible;
- 2 for its **local C -numerical range** $-1 \in W_{\text{loc}}(H, H)$;
- 3 its **local C -numerical range** is the interval
$$W_{\text{loc}}(H, H) = [-1; +1];$$



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3 H is locally unitarily similar to a \bar{H} with $\text{Ad}_{K_z}(\bar{H}) = -\bar{H};$

4 let $\mathfrak{g} = \mathfrak{g}_0 \oplus \bigoplus_{i \neq j} \mathbb{C} E_{ij}$ be the root-space

decomposition of $\mathfrak{sl}(N, \mathbb{C}); H$ is locally unitarily similar to a linear combination of root-space

elements to non-zero roots $\bar{H} := \sum_{\lambda=1}^m c_\lambda E_{ij}^{(\lambda)}$

satisfying a system of linear equations

$$\sum_\ell \rho_{\lambda,\ell} \cdot \phi_\ell = \pi \pmod{2\pi}$$



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Conclusions

"Show me the big effects!" Alexander PINES

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Conclusions & Outlook

1 Quantum Control: key in future technology

2 Timeoptimised Quantum Compilation

- dressed to physical hardware
- access to **time complexity**
- grossly fighting decoherence

3 Decoherence-Minimising Quantum Control

- progress towards controllability under dissipation

4 Local Time Reversal

- Hamiltonian simulation
- Hahn's spin echo by local controls

5 Local Optimisation

- pure-state entanglement
- new: local C -numerical range



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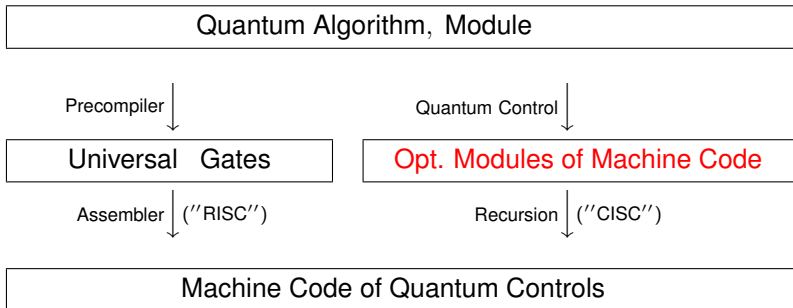


Outlook

Compilation by Recursion for Large Systems

- 0. Quantum Compiler
- I. Quantum Information
- II. Quantum Control
- III. Local Control
- Conclusions & Outlook

- recursive use of **optimised medium-sized building blocks**





Recursion

Ex.: Recursive QFT

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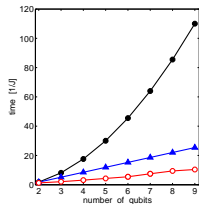
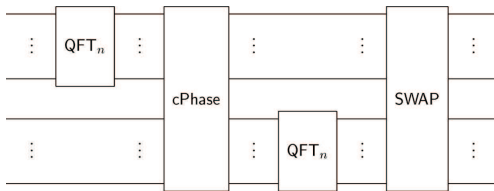
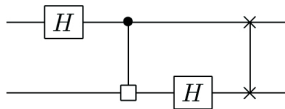
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III. Local Control

Conclusions & Outlook

■ **Principle:** from 2-qubit QFT to $2n$ -qubit QFT





Parallelisation

Speed-Up on High-Performance Parallel Cluster with T. Gradl, T. Huckle

0. Quantum Compiler

I. Quantum Information

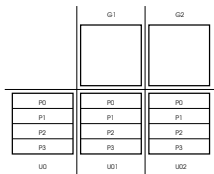
II. Quantum Control

III. Local Control

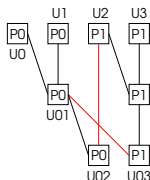
Conclusions & Outlook

■ Parallelising Matrix Operations

1. slice-wise:



2. tree-like:



■ Resulting Speed-Ups: 10 spins 128 time slices

128 AMD Opteron 850 CPU (2.4 GHz)

Subroutine	% of time	Speedup
optimizeCG	100	578
maxStepSize	90	709
getGradient	9.1	187
expm	7.5	879
propagation	1	31
gradient	0.6	81



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Frank Wilhelm, Markus Storcz, Johannes Ferber, Ville Bergholm
Tobias Gradl, Thomas Huckle
Navin Khaneja
Steffen J. Glaser

integrated EU programme QAP
initial steps: QIV

References:

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[math-ph/0701035](#), [math-ph/0702005](#)

0. Quantum
Compiler

I. Quantum
Information

II. Quantum Control

III. Local Control

Conclusions &
Outlook



2007-04-03

<http://www.informatik.tu-muenchen.de/ITEM?cmd=geta&aid=348>

#1

Stellenangebot

Studentische Hilfskraft für die Pflege und Wartung eines Softwarepakets "Parallele Matrixoperationen" auf dem HLRB-II Cluster im Rahmen eines EU-Projekts

Erforderliche Kenntnisse:

C++, MPI, UNIX/LINUX; Vordiplom

Nützlich:

Erfahrung mit Parallelrechner, Kenntnisse in Matlab

Projektbeschreibung:

Im Rahmen eines EU-Projekts: Pflege eines bestehenden Softwarepakets zur Lösung ambitionierter Optimierungsprobleme unter massiver Verwendung parallelisierter einfacher Matrixoperationen.

Die Programme laufen derzeit auf dem Infiniband-Cluster am Lehrstuhl Bode und am HLRB-II.

Dauer:

10-20 Std. pro Woche, 1 Semester mit Verlängerungsoption und Erfolgsprämie

Beginn:

Sofort oder sobald wie möglich

Kontakt:

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