Test exercises for the introductory seminar

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Problems 1 & 2 have been solved in class.

Problem 3:
Among grandmother's papers the following bill was found:

72 Döner....................-67,9- DM

The first and the last digit of the number which constituted the total price are no more readable and are represented by dashes here. What are the two erased digits and how much did each Döner (kebab) cost?

Problem 4:
Is it possible to draw 100 dots on different spots of the wooden cube such that each rotation of the cube that maps the cube to itself also maps the dots to themselves? How about 200 dots?
Hint: *every* non-interior point of the cube is eligible for drawing a dot.

Problem 5:
Otto has 44 one-euro coins and 10 pockets. He wishes to distribute his coins into his pockets in such a way that each pocket contains a different amount of money.

- Will he be able to do that?
- Solve the general problem assuming p pockets and n one-euro coins.

Problem 6:
A spider and a fly are moving along a fixed straight line (which has no end points). Each of the two animals has a constant absolute speed and can change its own moving direction at its will. The spider is twice as fast as the fly, but the spider knows nothing about the fly's location until they meet at the same point. Will the spider always be able to catch the fly?

Problem 7:
Two players alternately put bishops on free squares of an initially empty chess board, one bishop per player per turn, such that, ignoring the color, the bishops cannot directly attack each other. The player who can no more insert a bishop on his/her turn loses. Who wins: the starting player or his/her opponent?

Problem 8:
From a three-letter alphabet all possible words are constructed. During construction, certain combinations of two or more consecutive letters are considered forbidden. It is known that all forbidden combinations have different lengths. Do arbitrarily long allowed words exist?

Problem 9:
Students of a math course, identified by consecutive integers from 1 to n, are lined up in order $1, 2, \ldots, n-1, n$. On a command, each student can exchange places with someone else or stay where he/she is. Is it possible to get the order $n, 1, 2, \ldots, n-1$ as a result of two commands?

Problem 10:
On each of 44 trees growing in a circle a sparrow is sitting. From time to time, two arbitrary sparrows simultaneously fly over to their direct neighbor trees: one clockwise, another one counterclockwise. Will all the sparrows be able to gather on the same tree?

Problem 11:
Hundred different stones are placed in one row. Each two stones that are separated by exactly one stone can be interchanged. Is it possible to place all the stones in a reversed order only by such swapping?

Problem 12: Show that given n integers, we will inevitably find some of them (one or more) whose sum is divisible by n.

Problem 13: Twelve stars are written in a row: ************. Two players replace alternately a freely chosen star by a digit. If the obtained 12-digit number is divisible by 77, then the second player wins, otherwise the first one. Who wins: the starting player or his/her opponent?